



## INTRODUCTION

### Image Super-Resolution

- Conventionally, per-pixel  $L_p$  distances between HR images and SR images are used to train the deep image super-resolution network.
- Applying only the  $L_p$  distance leads to blurry results.

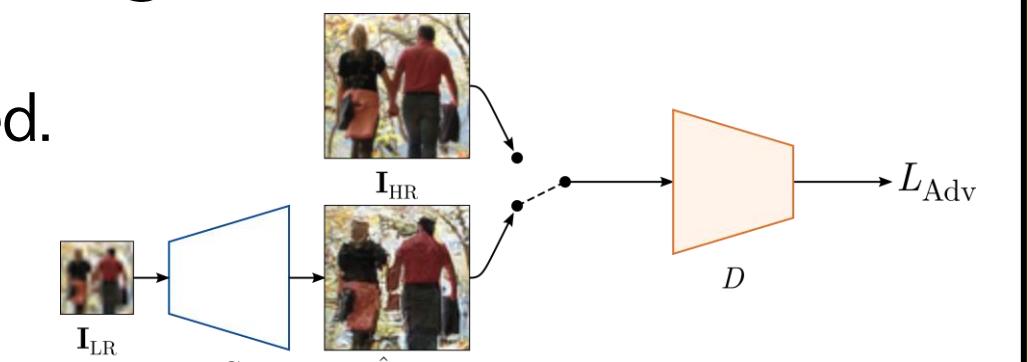


- More loss functions are designed by the computer vision community to alleviate the problem.

- Perceptual loss.
- Adversarial loss.

### Rethinking Adversarial Training

- In this scheme, the **generator** and the **discriminator** is alternatively trained.
- The structure of the **discriminator** is seldom discussed compared to that of the **generator**.
- Discriminator is an encoder:** it encodes **ground truth** or **generated** images to extract information of **realness** from them.
- Discriminator is a data model:** it models natural and generated images.
- Design of a data model** should be compatible to the **structure of the data** being modeled in order to achieve effective representation learning.
- Our goal** is to design a **discriminator** that is aware of the **structure of natural images** to provide more appropriate supervision in adversarial training framework – to be used in the image super-resolution problem.



### Structural Properties of Natural Images

- Three** the most salient structural features presented in natural images.
- These will guide the design process of our new **discriminator** model.

#### 1. Translation Equivariance:

Information in a natural image is concentrated (in the form of objects, textures, etc.) in a local region; translation only affects the positions and not the contents.

#### 2. Rotation Invariance:

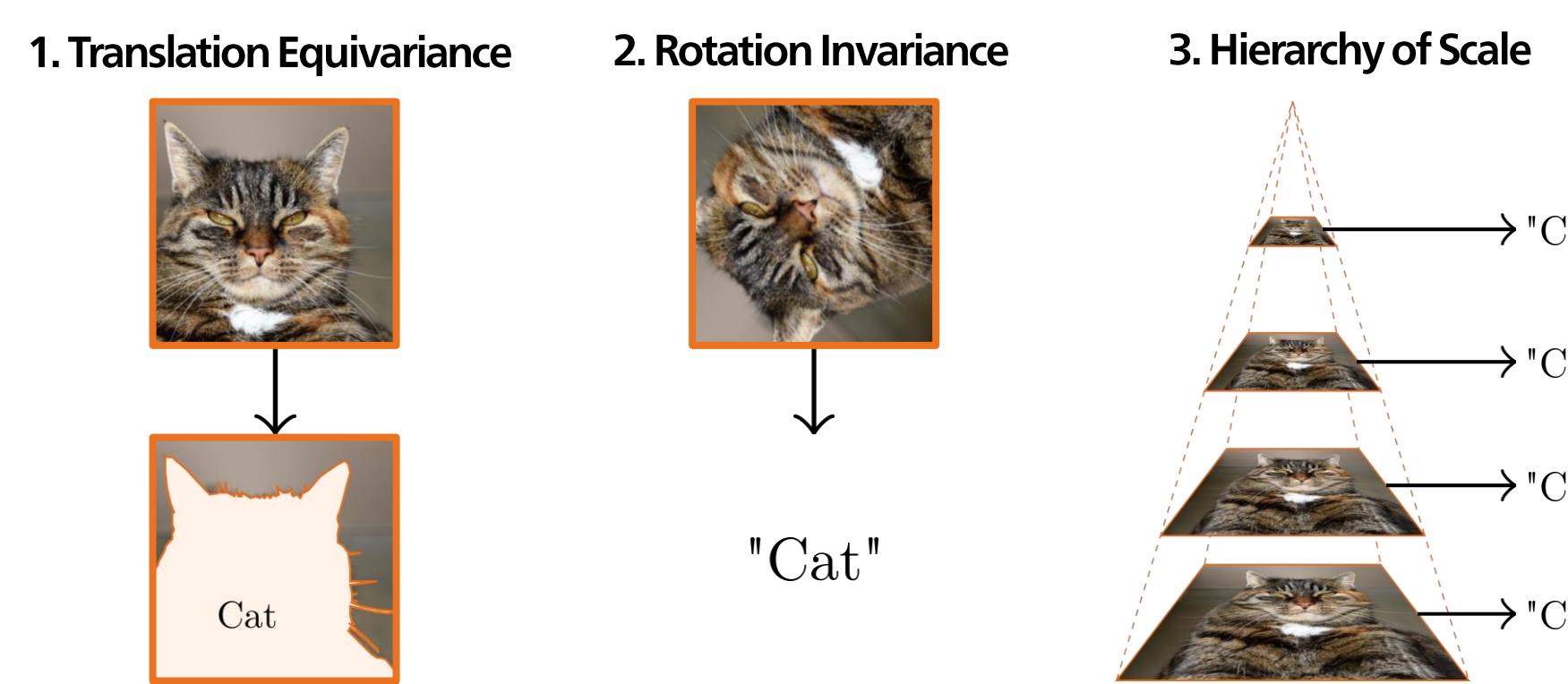
The same object can appear in arbitrary orientations.

#### 3. Hierarchy of Scale:

The same object can appear in various scales.

- These properties discusses about the **symmetries** occur in natural images.
- Symmetries** are represented in the language of **group theory** in mathematics; We will use the mathematical notion of **invariance** and **equivariance**.

## DESIGN



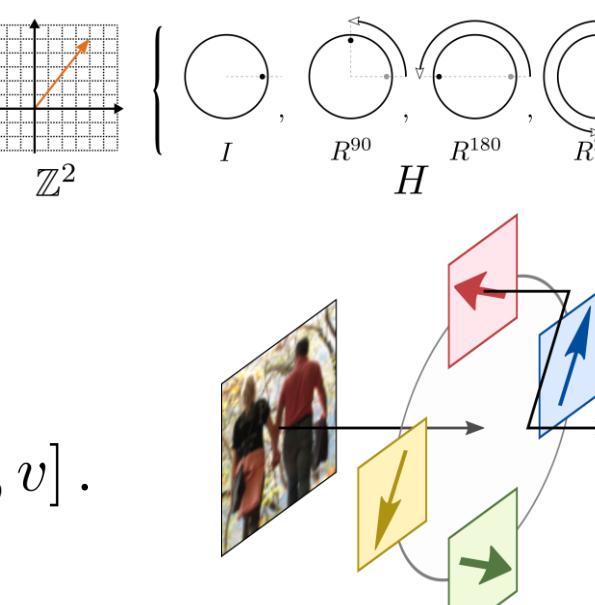
### 1. Translation Equivariance

- This property can be captured with a translation equivariant map:  $F' := f(F), f(\text{translate}(F, t)) = \text{translate}(F', t/s)$ .  $F \in \mathbb{R}^{b \times c \times h \times w}, F' \in \mathbb{R}^{b \times c \times h/s \times w/s}$ .
- A translation equivariant map
  - Ensures consistent **local** feature extraction.
  - Maintains **positional** information.

$p4 := \mathbb{Z}^2 \rtimes H$   
Roto-translation group  $p4$  is a semi-direct product of  
1. Discrete translation group  $\mathbb{Z}^2$   
2. Discrete 4-rotation group  $H$

### 2. Rotation Invariance

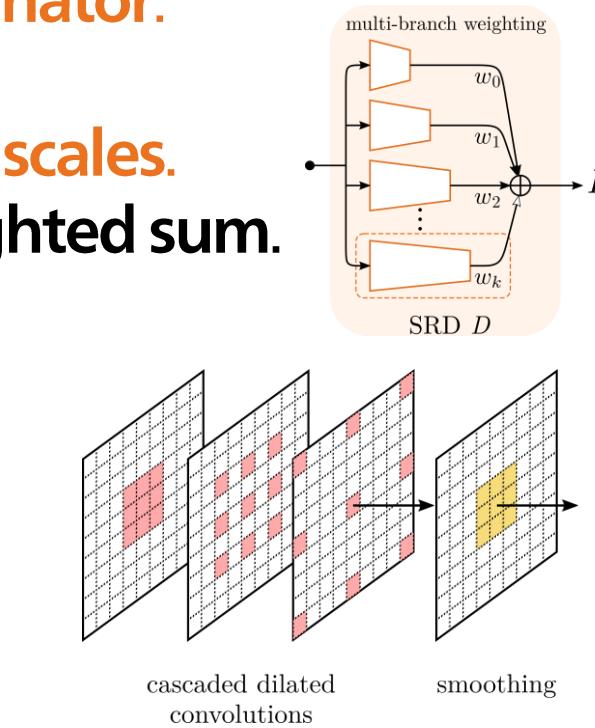
- Rotation invariance is embedded in the lowest layer of the **discriminator** where the gradients for the **generator** is generated from.
- It is built w.r.t. specific discrete roto-translation group,  $p4$ .
- Rotation invariant map is a composition of:
  - A discrete rotation equivariant convolution**  $[F']_\theta = [F *_{\mathbb{Z}^2 \rightarrow p4} k]_\theta := F * R^\theta k$ .  
 $F \in \mathbb{R}^{b \times c \times h \times w}, F' \in \mathbb{R}^{b \times c \times |H| \times h \times w}$ .
  - A rotation group-wise pooling layer**  
 $\text{gmaxpool}(F')[b, c, u, v] := \max_{g \in \mathcal{H}} F'[b, c, g, u, v]$ .  
 $\text{gmaxpool}(F') \in \mathbb{R}^{b \times c \times h \times w}$ .



### 3. Hierarchy of Scale

- The **scale** of an object  $\iff$  The **receptive field size** of a filter
- Two** design features are introduced to the **discriminator**:

  - Multi-branch architecture**
    - Multiple **receptive field sizes** for different **scales**.
    - Branch outputs are summarized by a **weighted sum**.
  - Cascaded dilated convolutions**
    - Efficient large receptive field by **exponentially increasing dilation**.
    - The **smoothing convolution** mitigates potential gridding artifact.



## IMPLEMENTATION

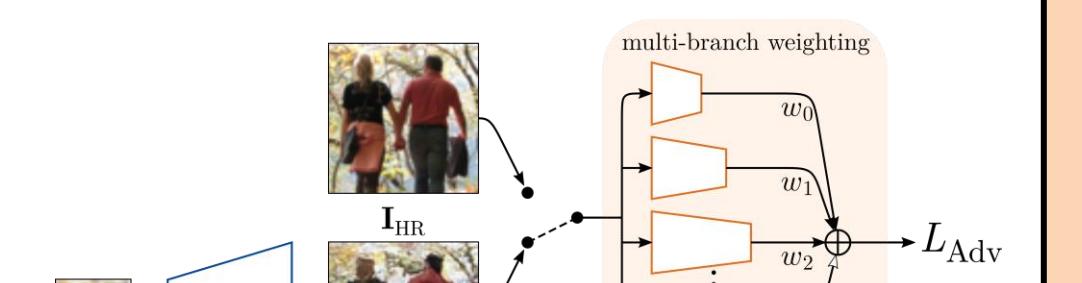
### Structure Resonant Discriminator (SRD)

- Multi-branch structure, each branch is a composition of:
  - Rotation invariant layer:**  $p4$ -equivariant conv + group-wise maxpool
  - Cascaded dilated convolutions** with exponentially increasing dilation
  - A single  $3 \times 3$  **smoothing convolution**
  - A  $1 \times 1$  convolution **classifier**

### Implementation Details

#### 1. Multi-branch architecture

branch ID $i$	depth $l$	receptive field size	branch weight $w_i$ (rel.)
1	1*	$25 \times 25$	1
2	2	$65 \times 65$	$1.25^2$
3	3	$129 \times 129$	$1.5^2$
4	4	$257 \times 257$	$1.75^2$
5	5	$513 \times 513$	$2^2$



#### 2. Cascaded dilated convolutions

- Enables exponential **receptive field size** w.r.t. the number of layers.

layer	gconv	conv1	conv2	dconv1	... dconv2	conv3	cls
kernel size	3	3	3	3	...	3	1
stride	1	2	2	1	...	1	1
dilation	1	1	1	2 <sup>l</sup>	...	1	1
receptive field size	3	+2	+4	+16	...	+2 <sup>l+3</sup>	+8



- Other loss functions**
  - L1 loss.**  $L_1(I_{HR}, \hat{I}_{HR}) = \mathbb{E}\|I_{HR} - \hat{I}_{HR}\|_1$ .
  - L1 perceptual loss.**  $L_{per}(I_{HR}, \hat{I}_{HR}) = \mathbb{E}\|\text{VGG}(I_{HR}) - \text{VGG}(\hat{I}_{HR})\|_1$ , Overall loss function:  $L_{tot} = \lambda_1 L_1 + \lambda_2 L_{per} + \lambda_3 L_{adv}$ .

### Training Details

- Loss function: LSGAN loss.**  $L_{adv} = 1/2 \mathbb{E}\|D(I_{HR}) - 1\|_2^2 + 1/2 \mathbb{E}\|D(I_{SR}) - 0\|_2^2$ ,  $L_{per} = 1/2 \mathbb{E}\|D(I_{SR}) - 1\|_2^2$ .
- SR Network: RRDB (ESRGAN baseline).**
- Dataset:** Mixture of DIV2K & Flickr2K.
- Optimizer:** Adam with default hyperparameter.

### Quantitative Results

Method	Set5			Set14			BSD100			Urban100		
	PSNR↑	SSIM↑	LPIPS↓	PSNR↑	SSIM↑	LPIPS↓	PSNR↑	SSIM↑	LPIPS↓	PSNR↑	SSIM↑	LPIPS↓
Bicubic	28.42	0.8104	0.340	26.00	0.7027	0.438	25.96	0.6675	0.524	23.14	0.6577	0.473
EDSR	32.46	0.8968	0.174	28.80	0.7876	0.284	27.71	0.7420	0.372	26.64	0.8033	0.232
RRDB	32.73	0.9011	0.173	28.99	0.7917	0.277	27.85	0.7455	0.366	27.03	0.8153	0.200
EDSR + GAN	29.39	0.8420	0.086	26.47	0.7183	0.146	25.73	0.6662	0.191	23.88	0.7169	0.155
SRGAN (Reproduced)	29.90	0.8486	0.081	26.56	0.7089	0.145	25.49	0.6524	0.177	24.38	0.7305	0.143
ESRGAN	30.44	0.8498	0.075	26.28	0.6981	0.134	25.28	0.6495	0.162	24.34	0.7327	0.123
SRGAN	28.93	0.817	0.087	25.43	0.678	0.162	-	-	-	-	-	-
RankSRGAN-NIQE	29.77	0.8363	0.073	26.48	0.7023	0.138	25.49	0.6484	0.177	24.53	0.7279	0.142
RankSRGAN-Ma	28.85	0.8204	0.078	25.79	0.6852	0.145	25.03	0.6390	0.185	24.12	0.7182	0.143
RankSRGAN-PI	29.65	0.8343	0.073	26.46	0.7021	0.137	25.44	0.6484	0.175	24.47	0.7289	0.138
EDSR + RaGAN + SRD (Ours)	29.91	0.8473	0.080	26.80	0.7187	0.134	25.78	0.6626	0.184	24.38	0.7326	0.149
RRDB + RaGAN + SRD (Ours)	30.46	0.8523	0.074	26.73	0.7129	0.126	25.80	0.6663	0.164	24.72	0.7441	0.124
RRDB + LSGAN + SRD (Ours)	30.55	0.8506	0.063	26.93	0.7227	0.126	25.87	0.6693	0.157	25.07	0.7542	0.117

Method	DIV2K-Val			PIRM-Val			PIRM-Test			OST100		
PSNR↑	SSIM↑	LPIPS↓										



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